Funções trignométricas e suas inversas

Cálculo I

Função Inversa

Definição

Uma função $f^{-1}B \to D$ diz-se inversa de $f : D \to B$ se satisfaz:

$$f \circ f^{-1}(x) = f^{-1} \circ f(x) = I(x)$$

onde $I(x) = x$ é a função identidade.

Uma função é invertível se e só se é bijectiva.

$$f : D \to B \qquad\qquad f^{-1} : B \to D$$

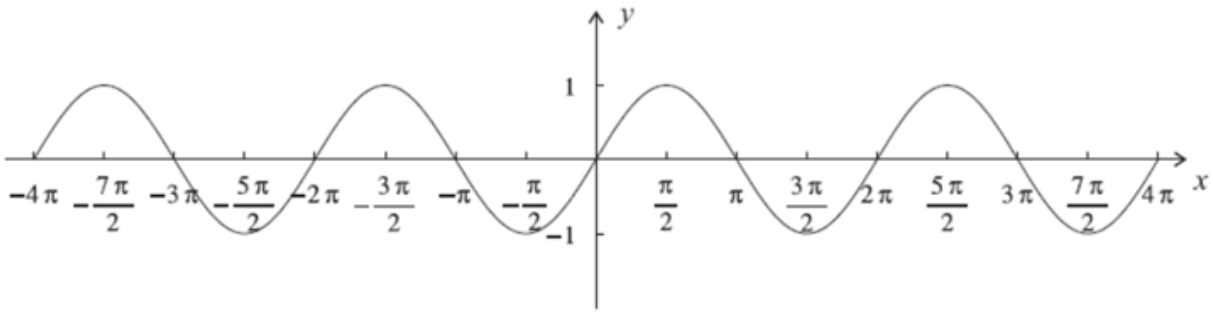
$$x \mapsto f(x) \qquad\qquad y \mapsto f^{-1}(y)$$

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

▸ Exemplo

Função Seno

$$f : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto \operatorname{sen}(x)$$



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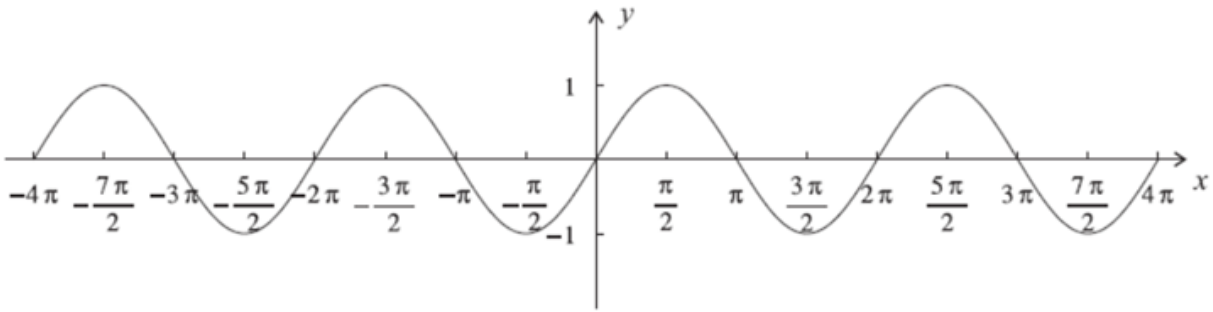

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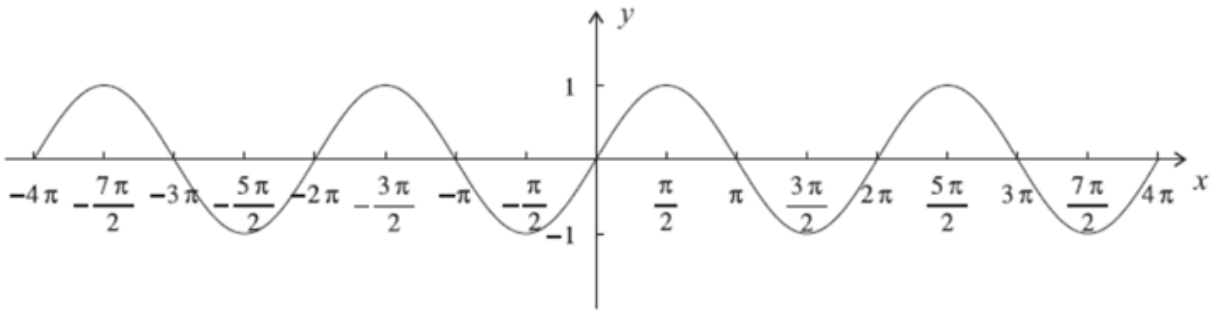

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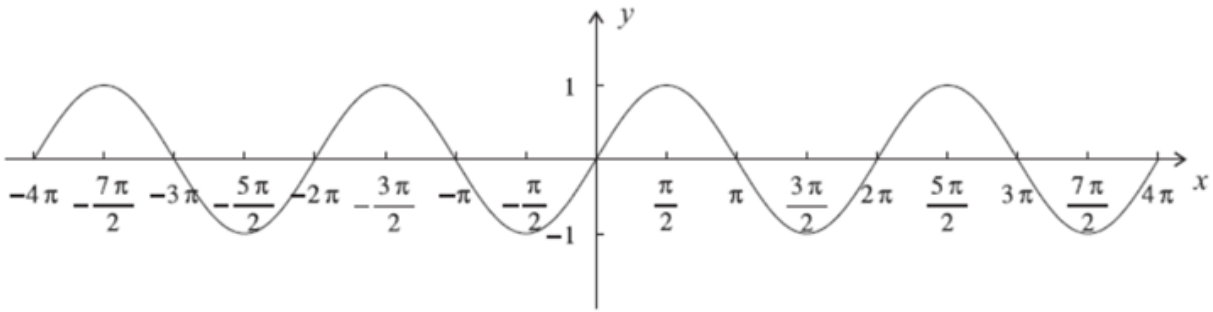

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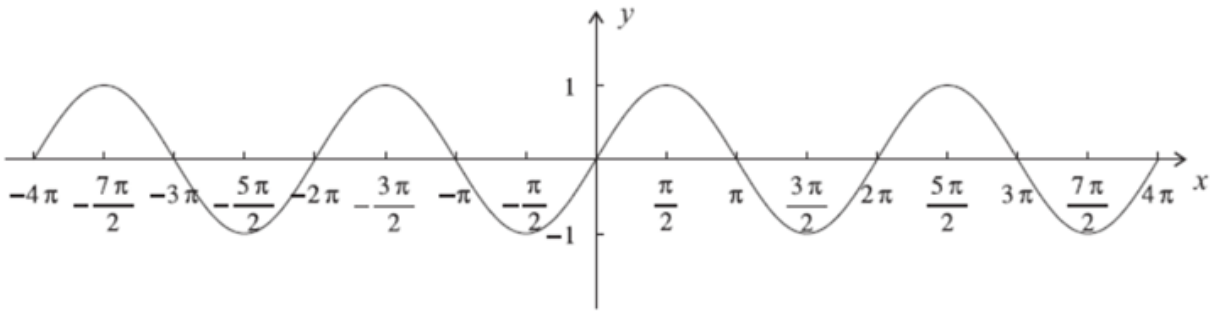

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- **Restrição principal do seno:**

$$f : [-\tfrac{\pi}{2}, \tfrac{\pi}{2}] \to [-1, 1]$$
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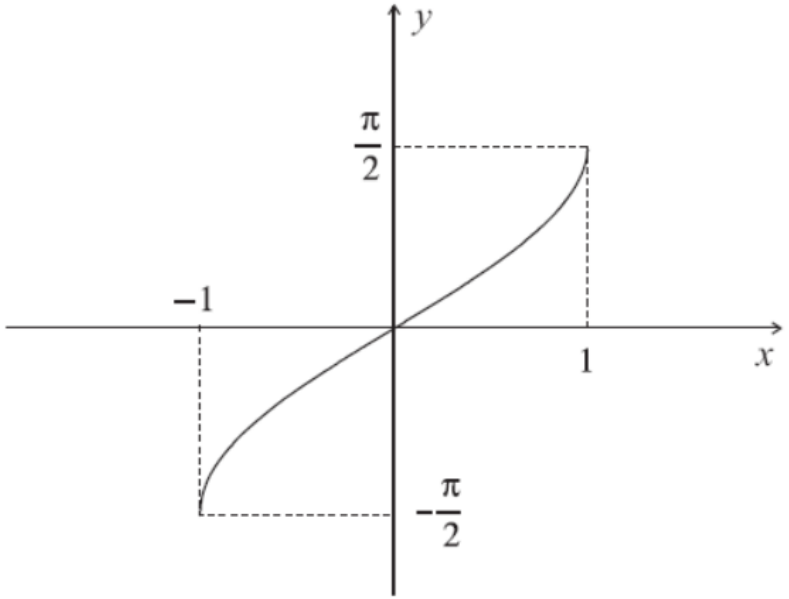
Função Arcoseno



$$f^{-1} : [-1, 1] \to [-\tfrac{\pi}{2}, \tfrac{\pi}{2}]$$
$$x \mapsto \operatorname{arcsen}(x)$$

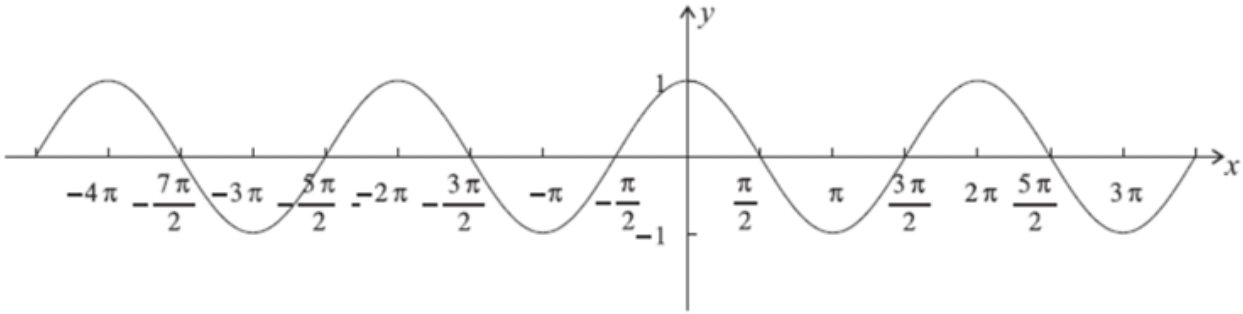
$$\forall y \in [-\tfrac{\pi}{2}, \tfrac{\pi}{2}], \quad x \in [-1, 1] :$$

$$x = \operatorname{sen}(y) \Leftrightarrow y = \operatorname{arcsen}(x)$$

Exemplos

Função Coseno

$$f : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto \cos(x)$$



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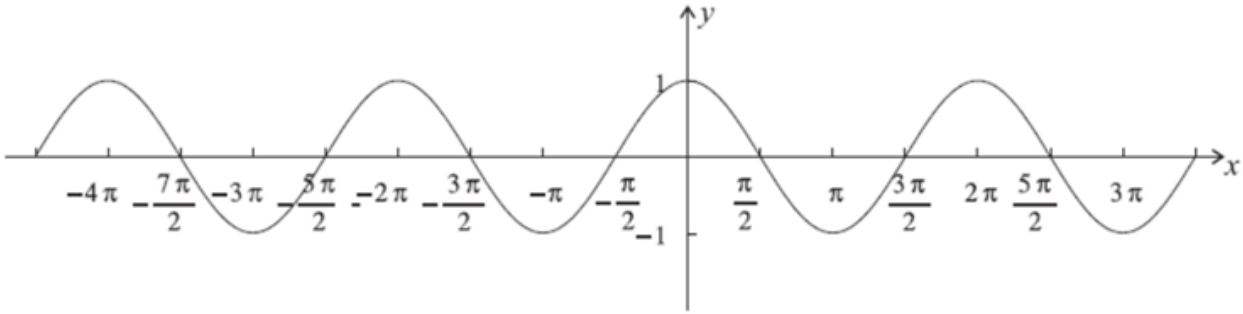

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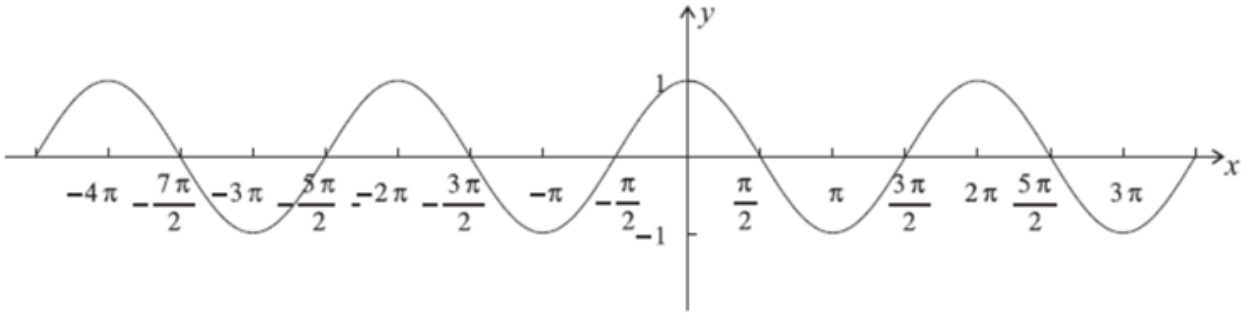

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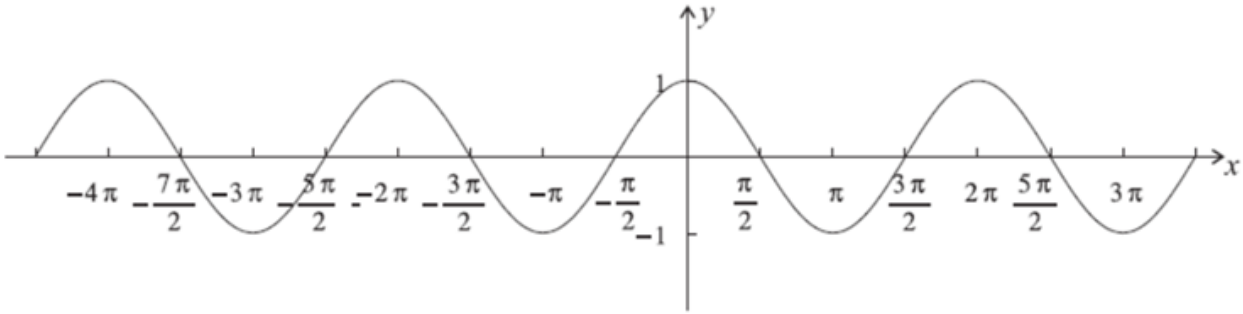

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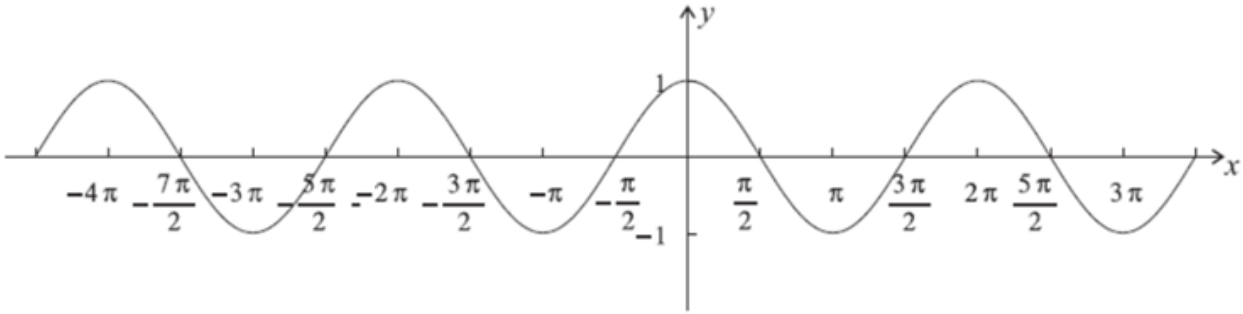

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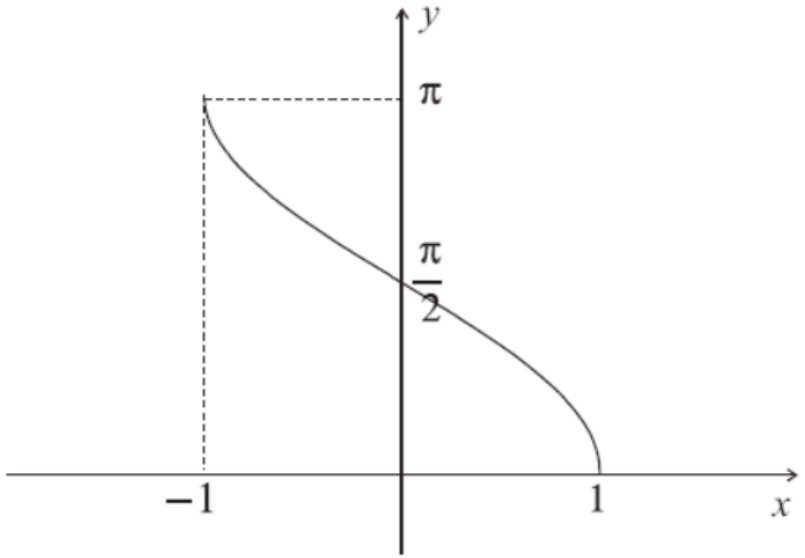
Função Arcocoseno



$$f^{-1} : [-1, 1] \to [0, \pi]$$
$$x \mapsto \arccos(x)$$

$$\forall y \in [0, \pi], \quad x \in [-1, 1] :$$

$$x = \cos(y) \Leftrightarrow y = \arccos(x)$$

Exemplos

Função Tangente

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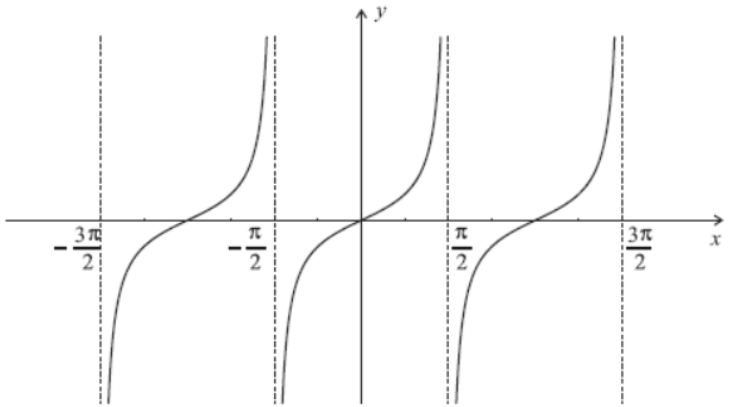

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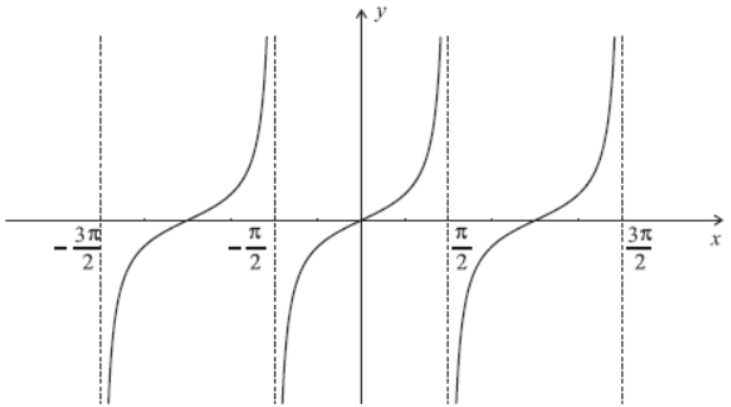

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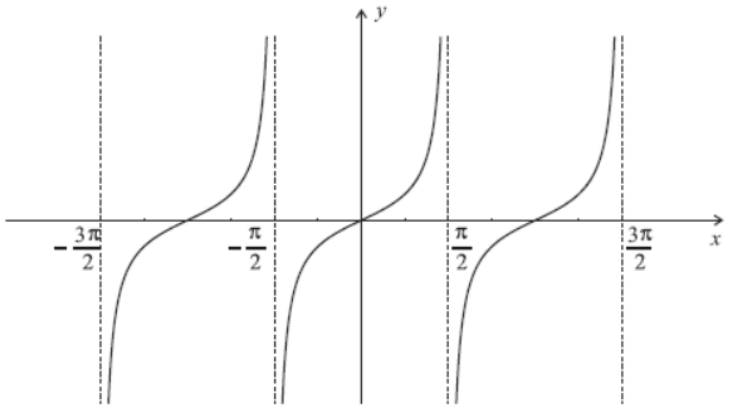

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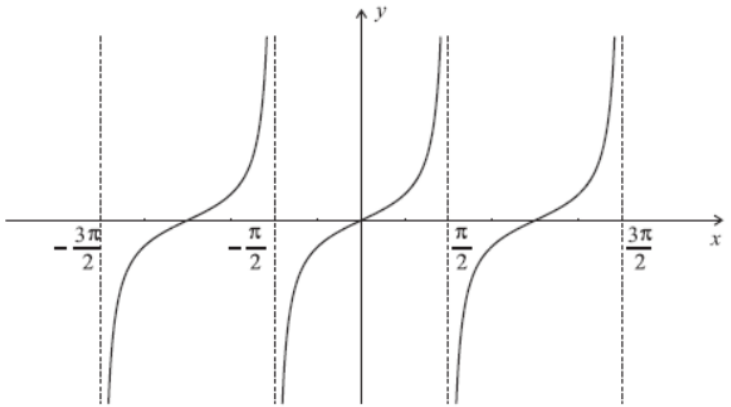

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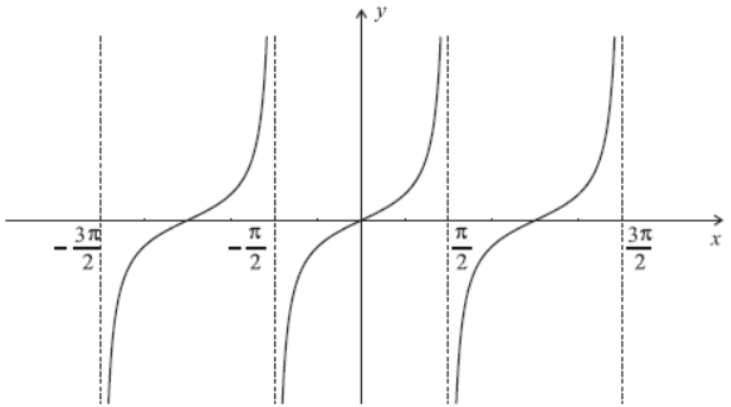

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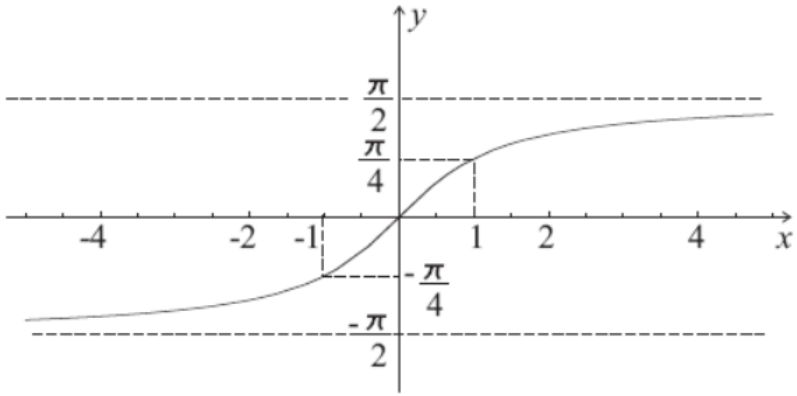

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Exemplos

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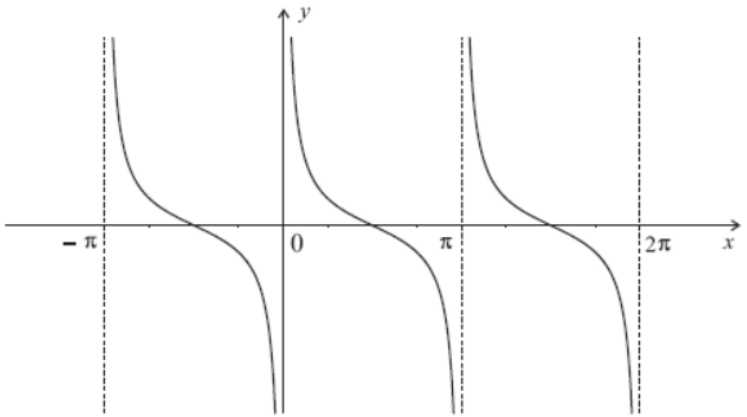
$$x \mapsto \operatorname{cotg}(x) = \frac{\cos(x)}{\operatorname{sen}(x)}$$



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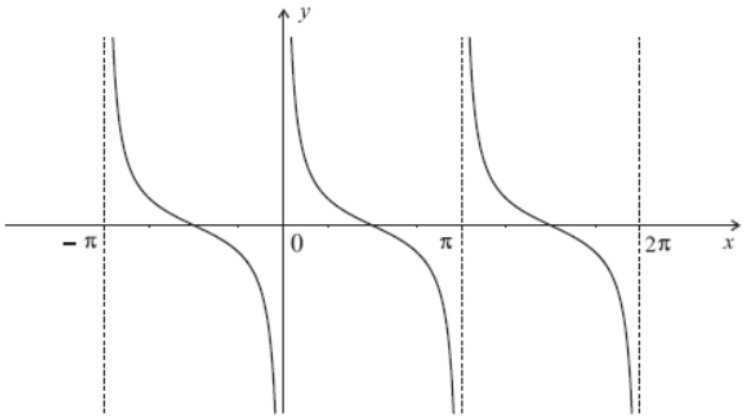

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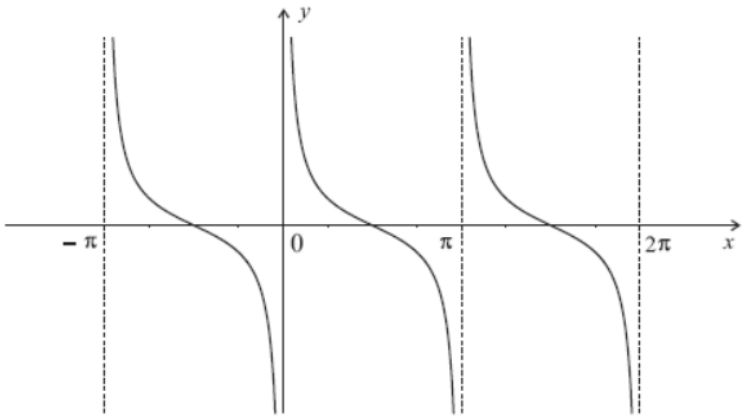

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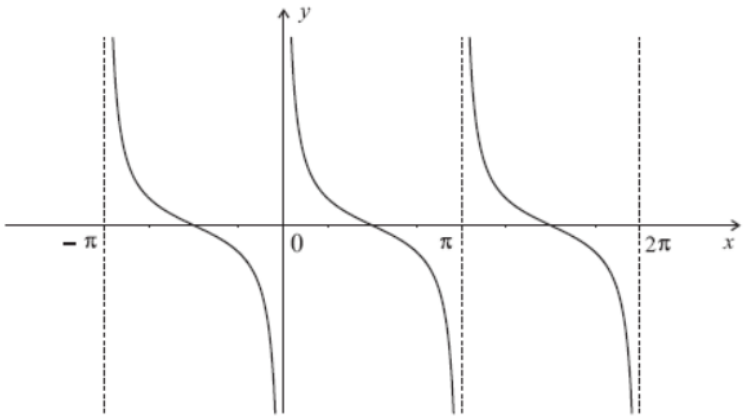

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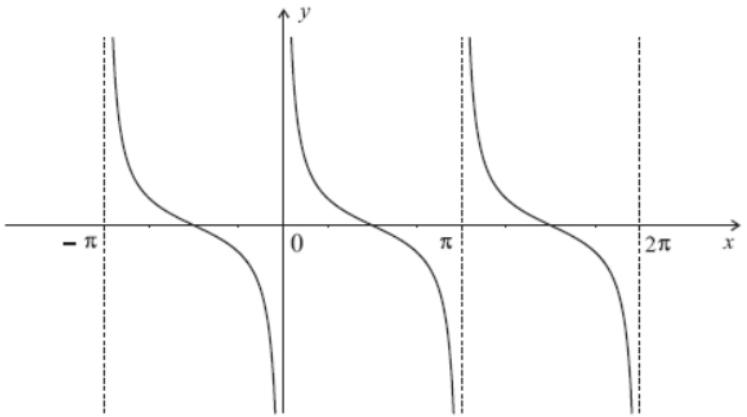

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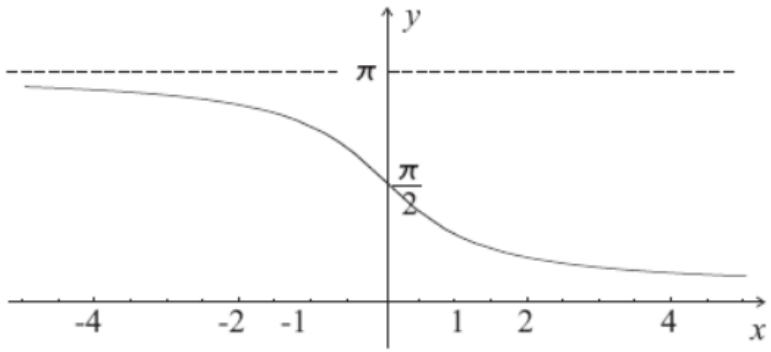

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